B Trees

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2020/11/02 @ TR-313, NTUST

Schedule

- Midterm exam will be held at 11/16 (Mon.)
  - Homework 2 will be announced at 11/9 (Mon.)
  - 11/11 (Wed.) is our study holiday!

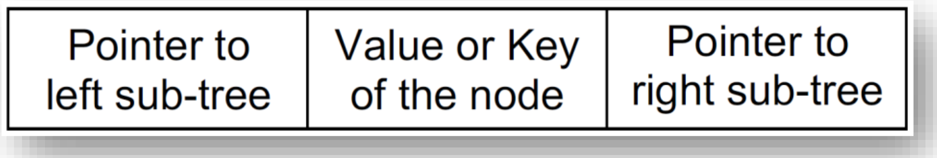
Review

- Splay tree is a **self-balancing** and a **self-optimizing** data structure
  - A simple idea behind it is that if an element is accessed, it is likely that it will be accessed again
    - The frequently accessed nodes are moved closer to the root so that they can be accessed quickly

- Self-balancing binary search trees
  - AVL Tree
  - Red-black Tree
  - Splay Tree

Multi-way Search Trees.

- Every node in a binary search tree contains one value and two pointers, left and right, which point to the node’s left and right sub-trees

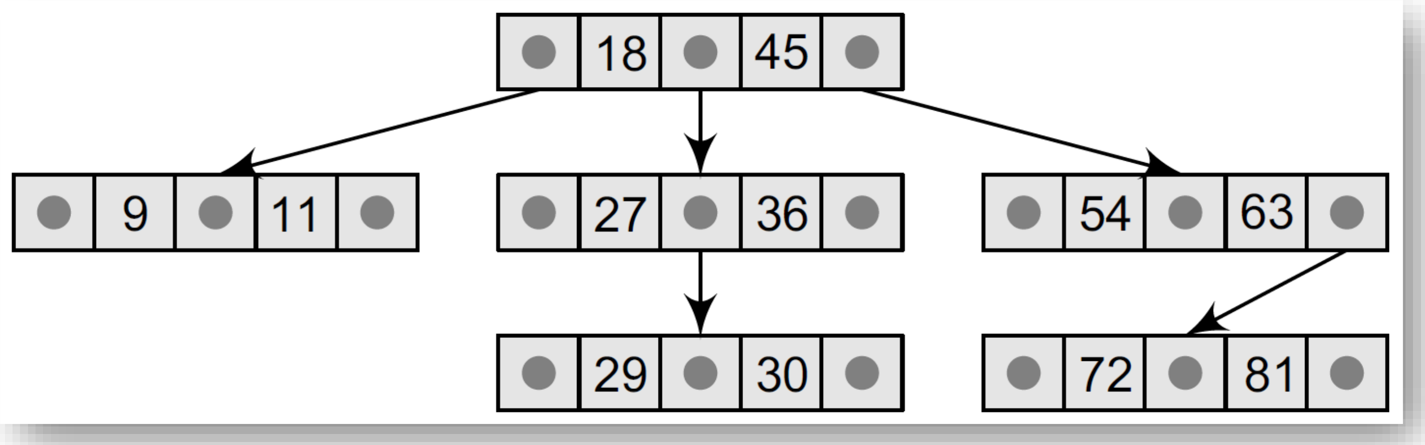


Pointer to left sub-tree	Value or Key of the node	Pointer to right sub-tree

- An M-way search tree has $M-1$ values per node and M subtrees (pointers)
  - M is called the degree of the tree
  - If $M=2$, each node in the M-way search tree has one value and two sub-trees
    - Binary Search Tree!

Multi-way Search Trees..

- For a M-way search tree
  - All the key values are stored in ascending order
    - 3-way search tree



  - It is not compulsory that every node has exactly M–1 values and M subtrees
    - The node can have anywhere from 1 to M–1 values
    - The number of sub-trees can vary from 0 (leaf node) to M

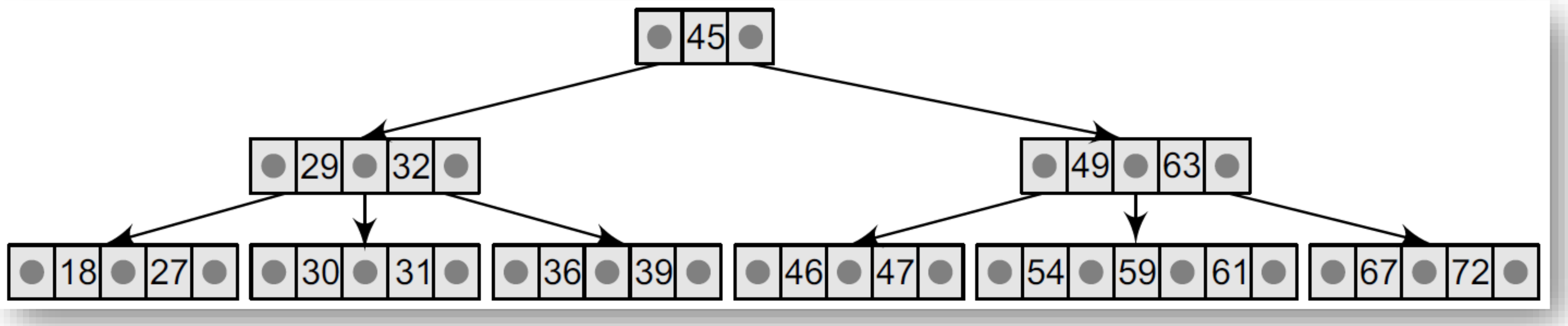
B Trees.

- A B tree is a specialized M-way tree developed by Rudolf Bayer and Ed McCreight in 1970
  - A B tree of order m can have a maximum of $m-1$ keys and m pointers to its sub-trees

- A B tree of order m is a tree with all the properties of an M-way search tree and has additional properties
  - Every node in the B tree has at most (maximum) m children
  - Every node in the B tree except the root node and leaf nodes has at least (minimum) $\left\lceil \frac{m}{2} \right\rceil$ children
    - Degree=4, at least 2 children, at least 1 key
    - Degree=5, at least 3 children, at least 2 key
  - The root node has at least two children
  - All leaf nodes are at the same level

B Trees..

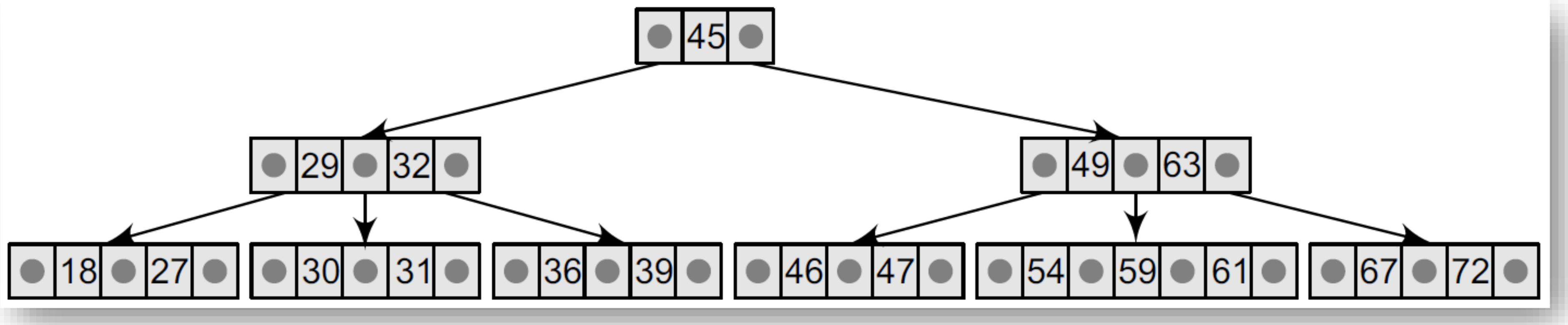
- An example of B tree, whose order is 4



  - Degree=4, at least 2 children, at least 1 key
  - The root node has at least two children
  - All leaf nodes are at the same level

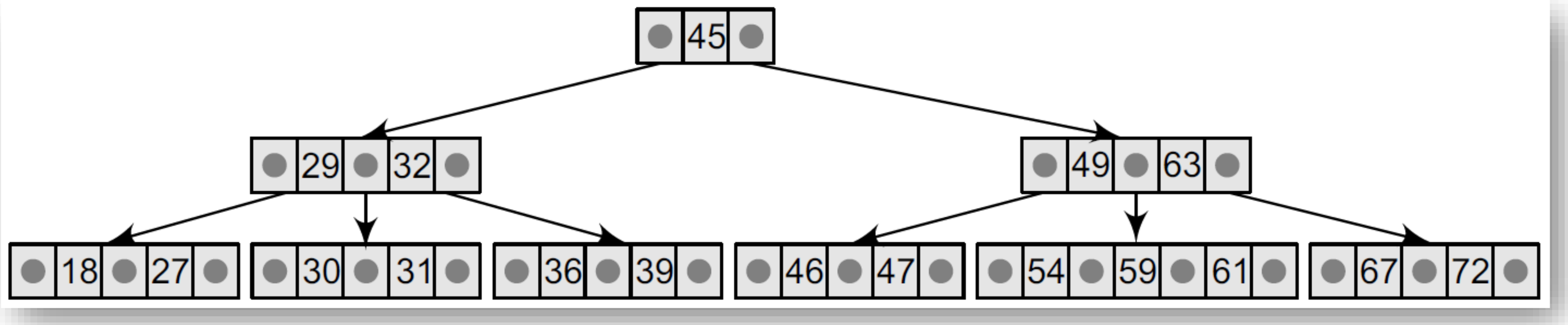
- While performing insertion and deletion operations in a B tree, the number of child nodes may change
  - The internal nodes may be **joined** or **split** to maintain a minimum number of children

Searching in a B Tree.



- To search for 59
  - The root node has a value 45 which is less than 59
    - Go to right sub-tree
  - The right sub-tree of the root node has two key values, 49 and 63
    - Since 49 < 59 < 63, traverse the right sub-tree of 49, or the left sub-tree of 63
  - This sub-tree has three values, 54, 59, and 61
    - Terminal

Searching in a B Tree..



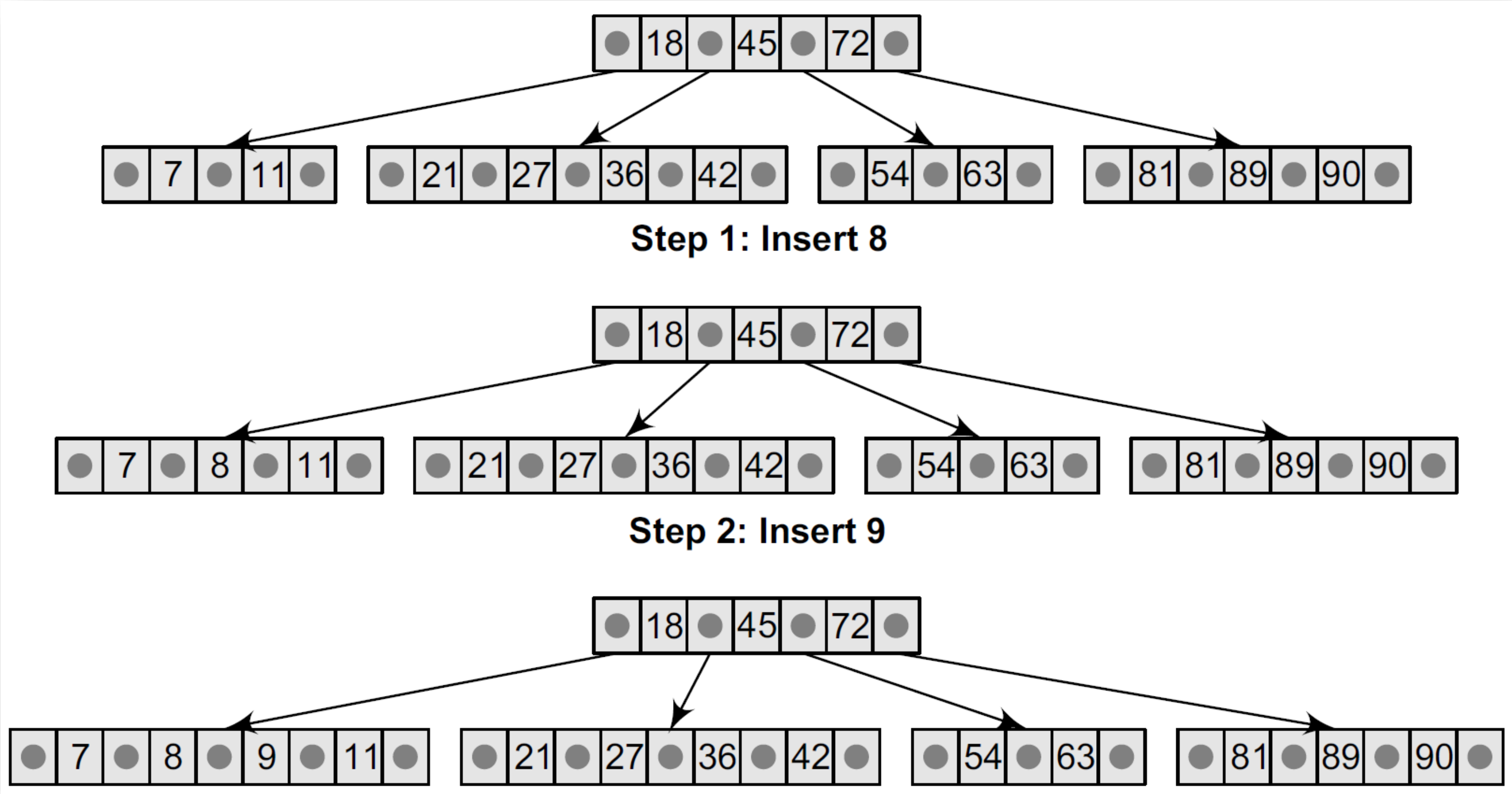
- To search for 9
  - Traverse the left sub-tree of the root node
  - The left sub-tree has two key values, 29 and 32
    - Traverse the left sub-tree of 29
  - The sub-tree has two key values, 18 and 27
    - There is no left sub-tree of 18
    - The value 9 is not stored in the tree

Inserting a New Element

- In a B tree, **all insertions are done at the leaf node level**
  1. Search the B tree to find the leaf node where the new key value should be inserted
  2. If the leaf node is not full
     - Insert the new element in the node keeping the node's elements ordered
  3. If the leaf node is full
     - Insert the new value in order into the existing set of keys
     - Split the node at its median into two nodes
       - ➢ The split nodes are half full
     - Push the median element up to its parent's node
       - ➢ If the parent's node is not full
         - □Done!
       - ➢ If the parent's node is already full
         - □Split the parent node by the same steps

Example.

- Given a B tree of order 5, please insert 8, 9, 39, and 4 into it
  - Degree=5, at least 2 keys & 3 children



Step 1: Insert 8

Step 2: Insert 9

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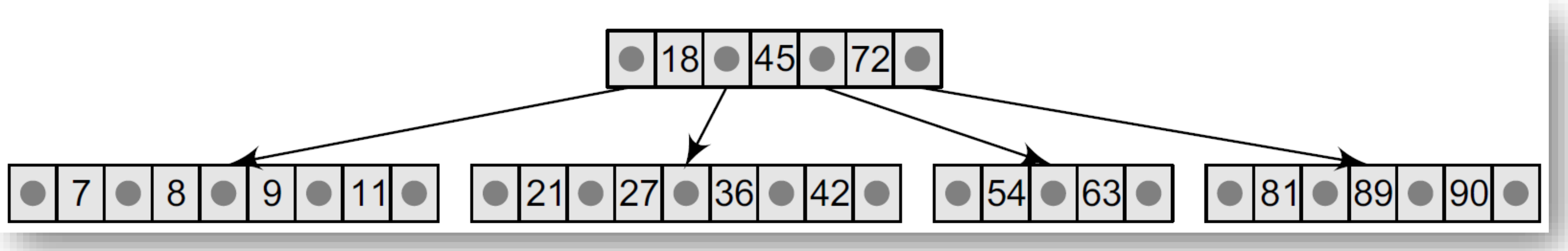
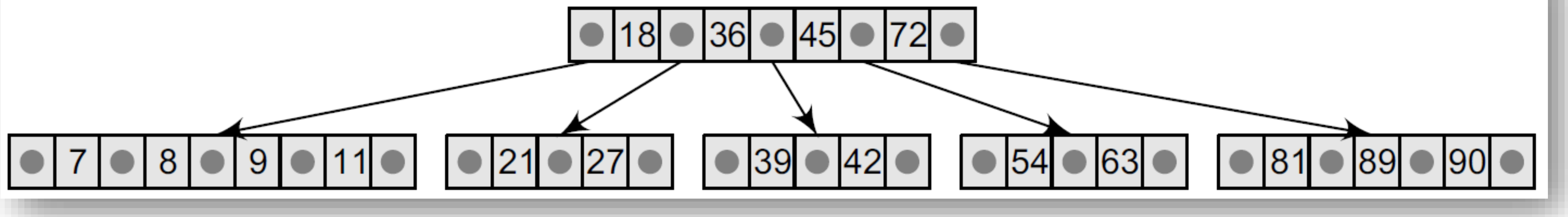


Step 3: Insert 39



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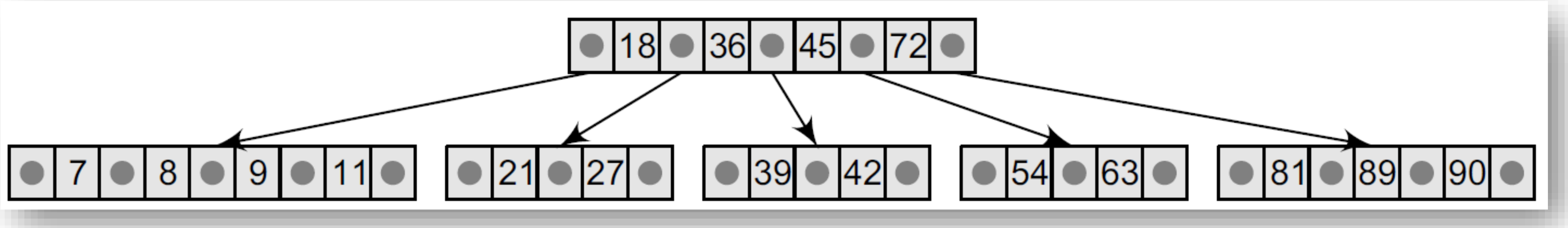
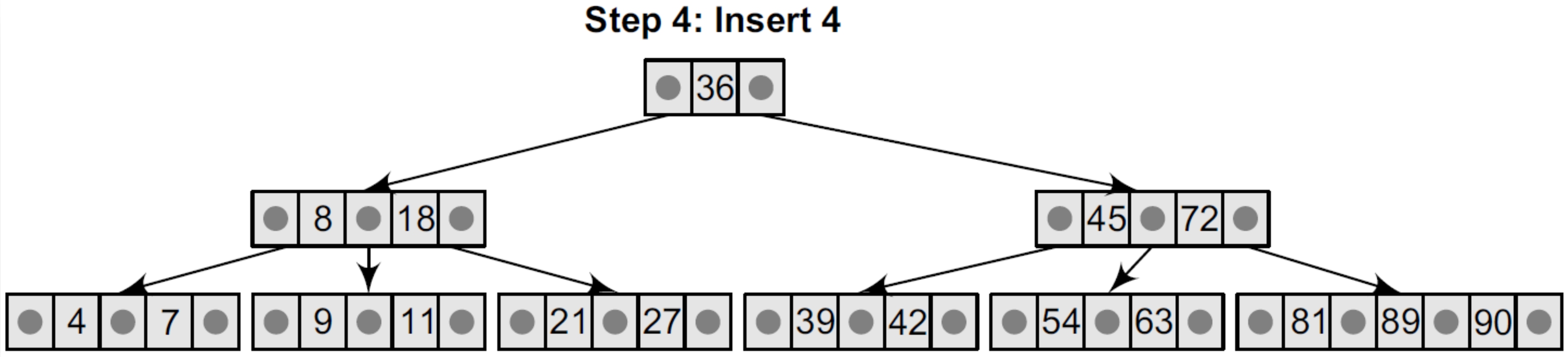




Deleting a New Element.

- There are two cases of deletion
  - A leaf node has to be deleted
  - An internal node has to be deleted
    - Promote the successor or predecessor of the key **in the leaf node** to occupy the position of the deleted key
      - ➢ The processing will be done as if a value from the leaf node has been deleted

Deleting a New Element..

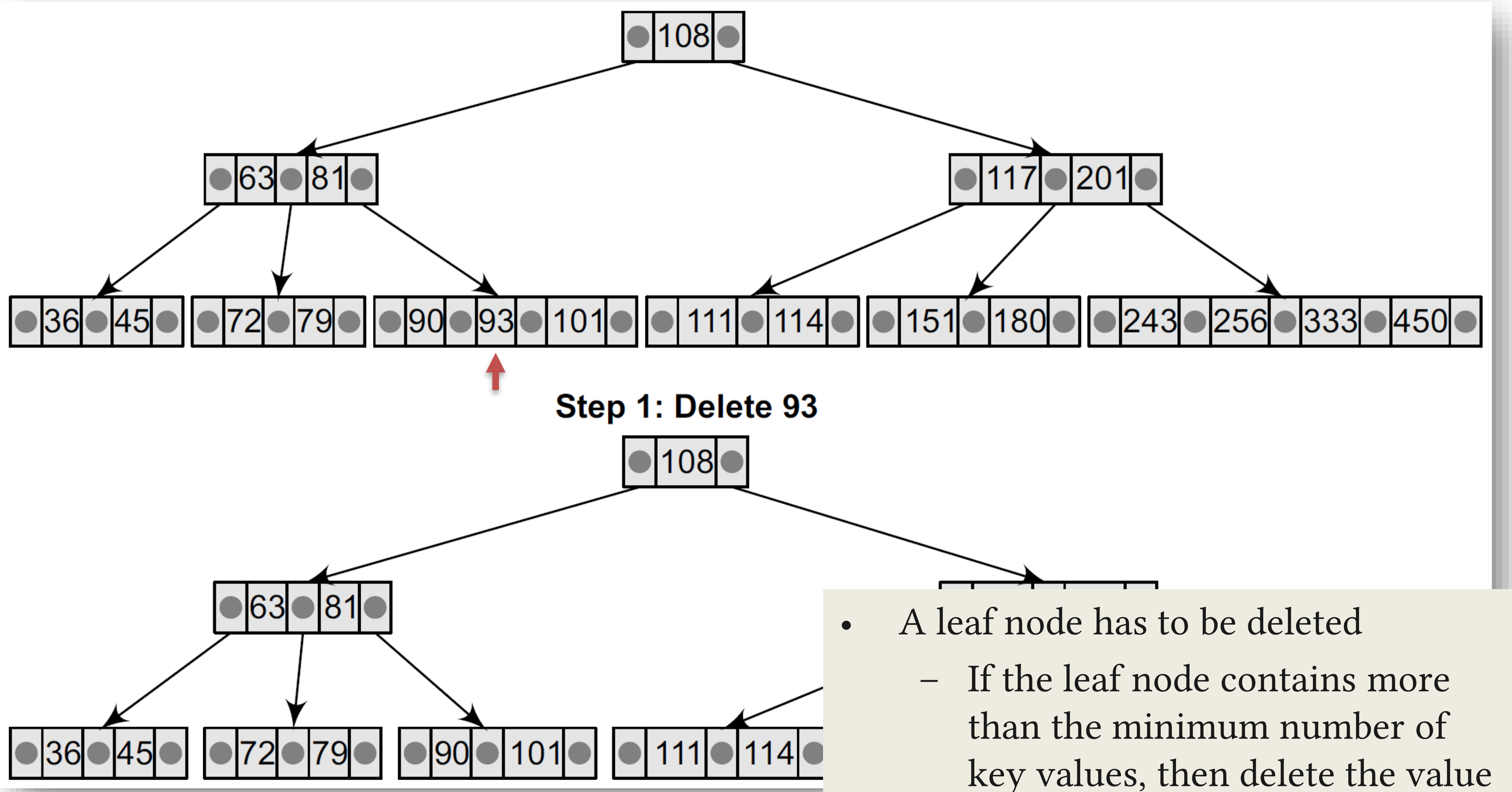
- A leaf node has to be deleted
  - Locate the leaf node which has to be deleted
  - If the leaf node contains more than the minimum number of key values, then delete the value
  - If the leaf node does not contain the minimum number elements, then fill the node by **taking an element either from the left or from the right sibling**
    - ➢ If the left sibling has more than the minimum number of key values
      - □ push its largest key into its parent's node
      - □ pull down the suitable (intervening) element from the parent node to replace the deleted element
    - ➢ If the right sibling has more than the minimum number of key values
    - ➢ If both left and right siblings contain only the minimum number of elements

Deleting a New Element...

- If the left sibling has more than the minimum number of key values
- If the right sibling has more than the minimum number of key values
  - push its smallest key into its parent’s node
  - pull down the suitable (intervening) element from the parent node to replace the deleted element
- If both left and right siblings contain only the minimum number of elements
  - create a new leaf node by combining the two leaf nodes (target+left or target+right) and the intervening element of the parent node
  - if the parent node contains less than the minimum number of keys in the node
    - ✓ propagate the process upwards, thereby reducing the height of the B tree

Example – 1.

- Given a B tree of order 5, please delete 93, 201, 180, and 72
  - Degree=5, at least 3 children & 2 keys



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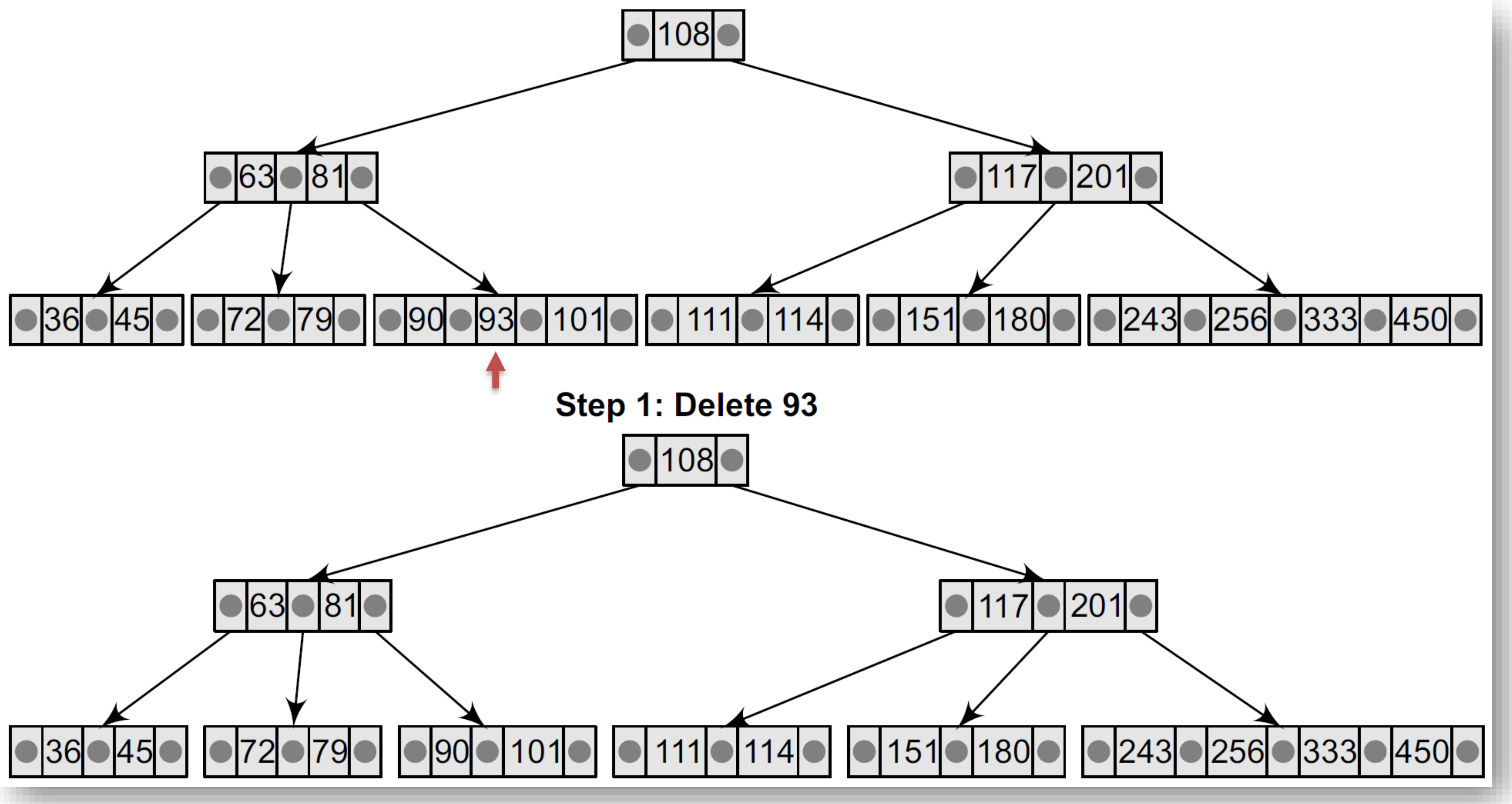

Step 1: Delete 93

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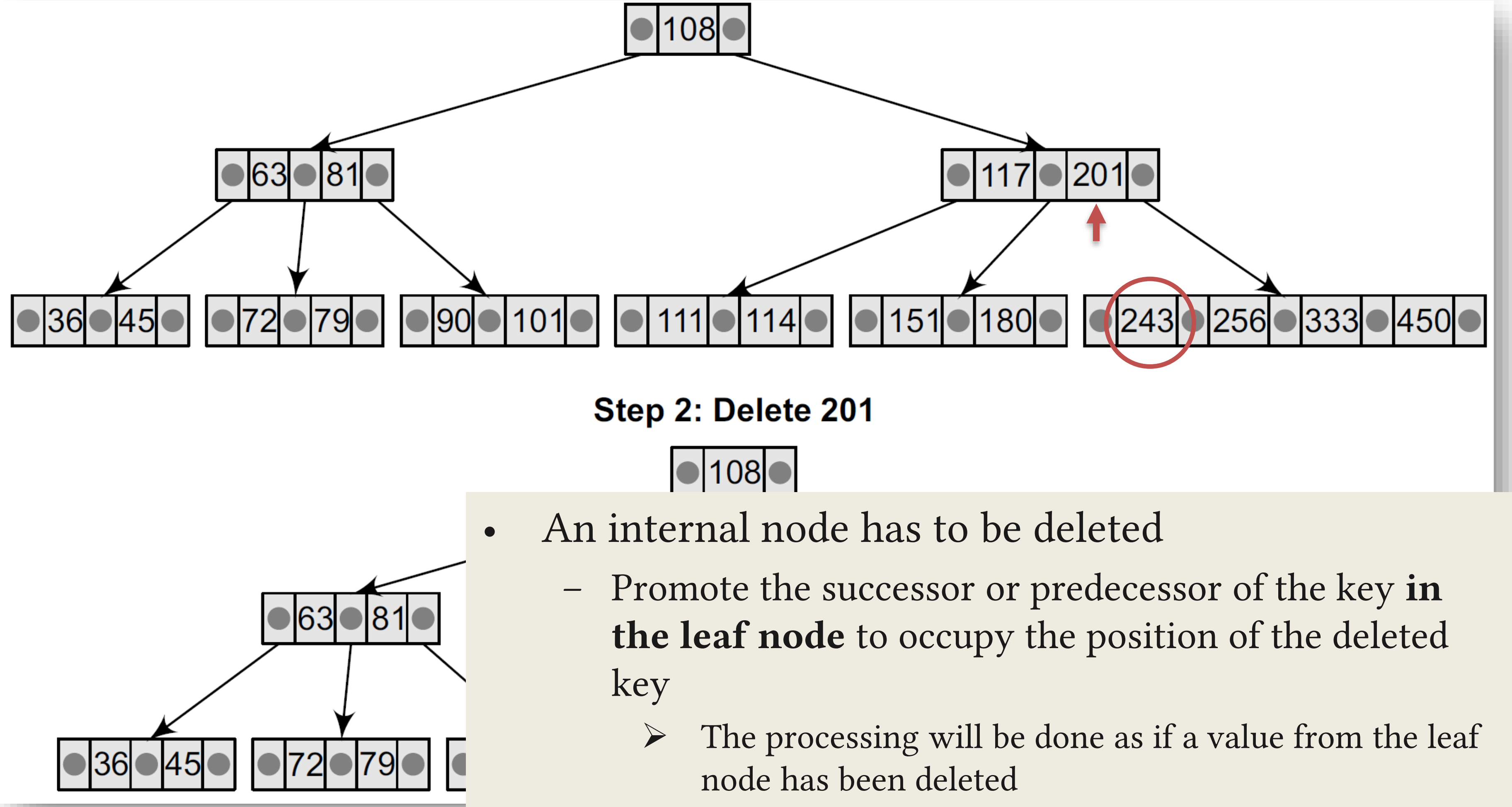

Step 2: Delete 201

- An internal node has to be deleted
  - Promote the successor or predecessor of the key **in the leaf node** to occupy the position of the deleted key
    - ➢ The processing will be done as if a value from the leaf node has been deleted

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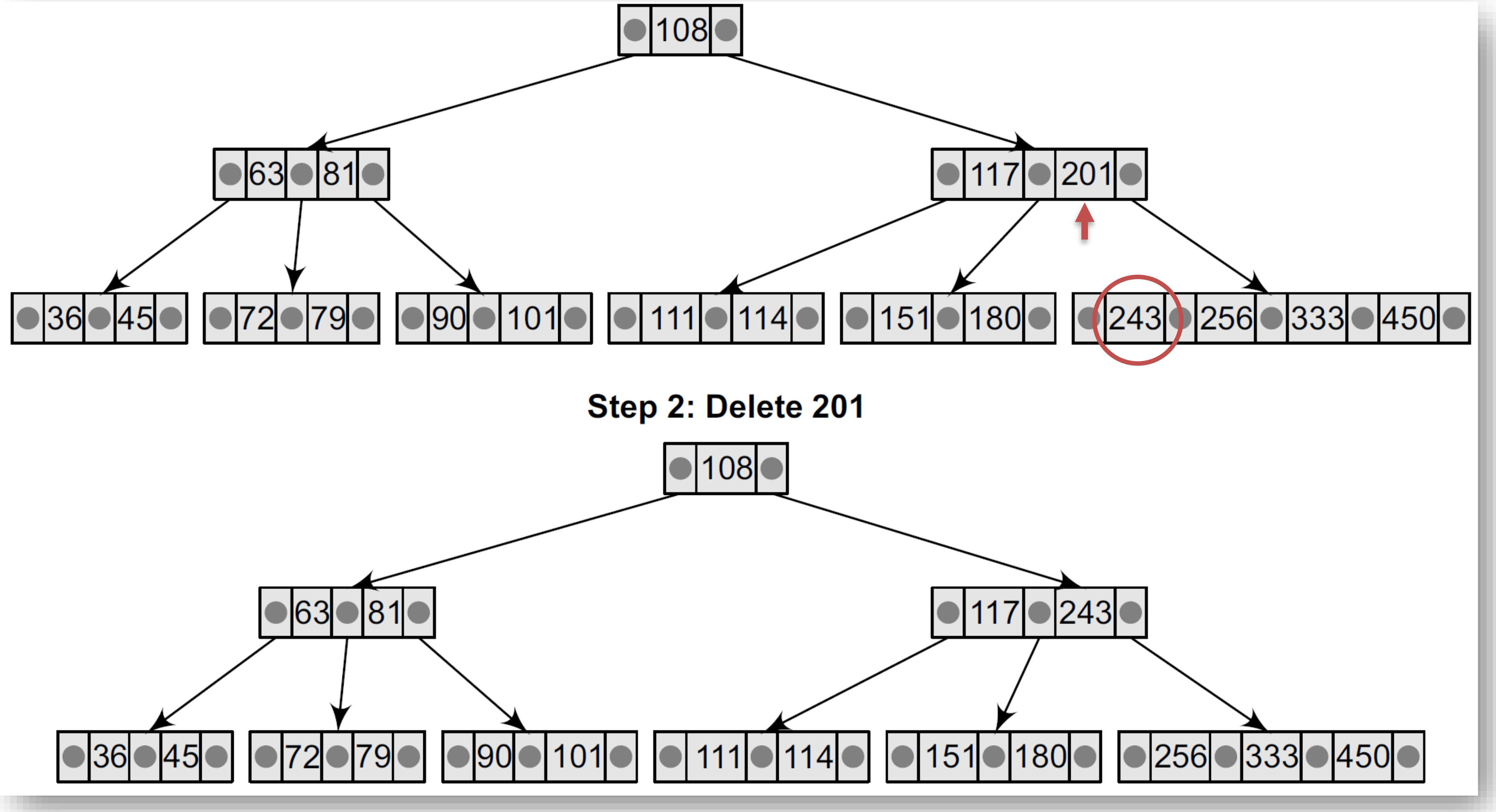

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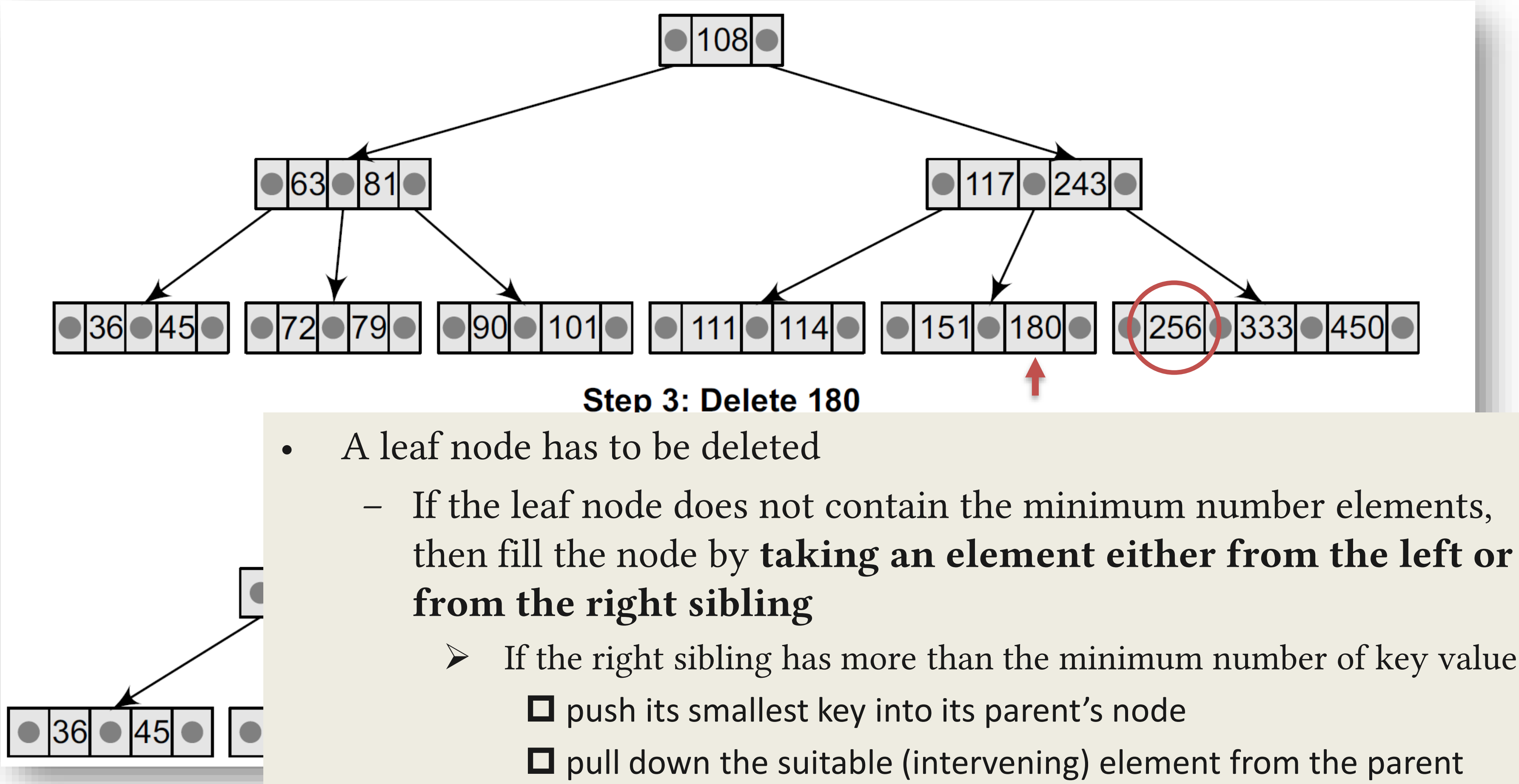

Step 3: Delete 180

- A leaf node has to be deleted
  - If the leaf node does not contain the minimum number elements, then fill the node by **taking an element either from the left or from the right sibling**
    - ➢ If the right sibling has more than the minimum number of key values
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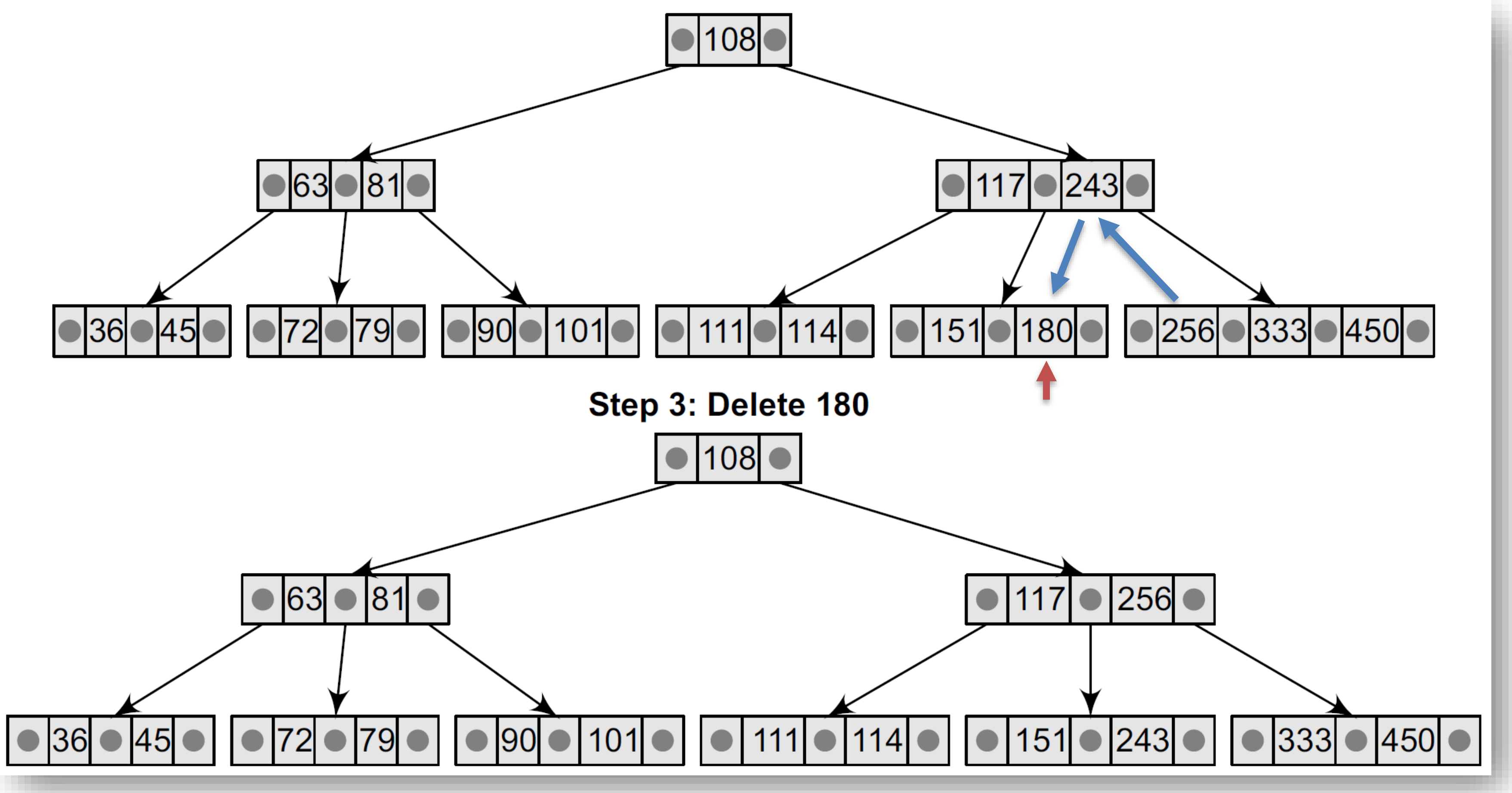

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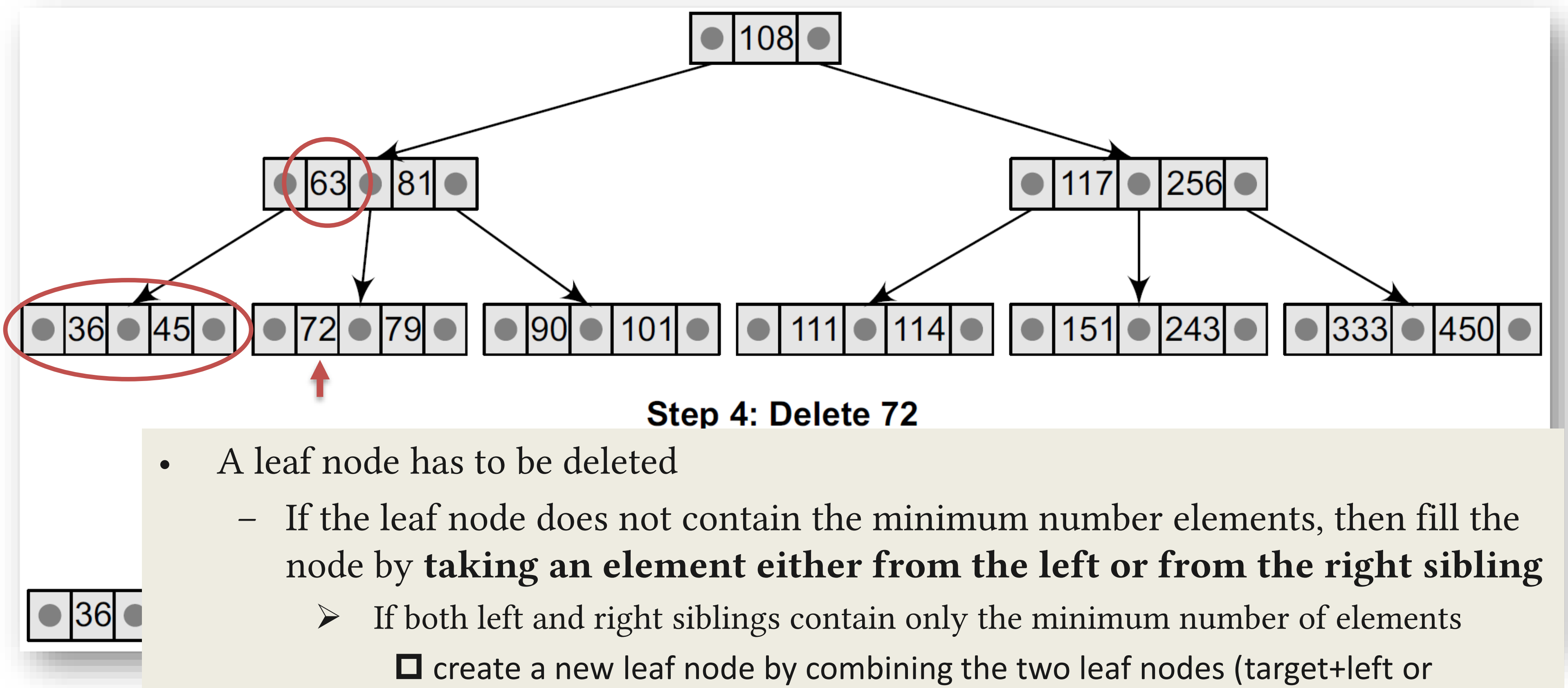

Step 4: Delete 72

- A leaf node has to be deleted
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    - ➢ If both left and right siblings contain only the minimum number of elements
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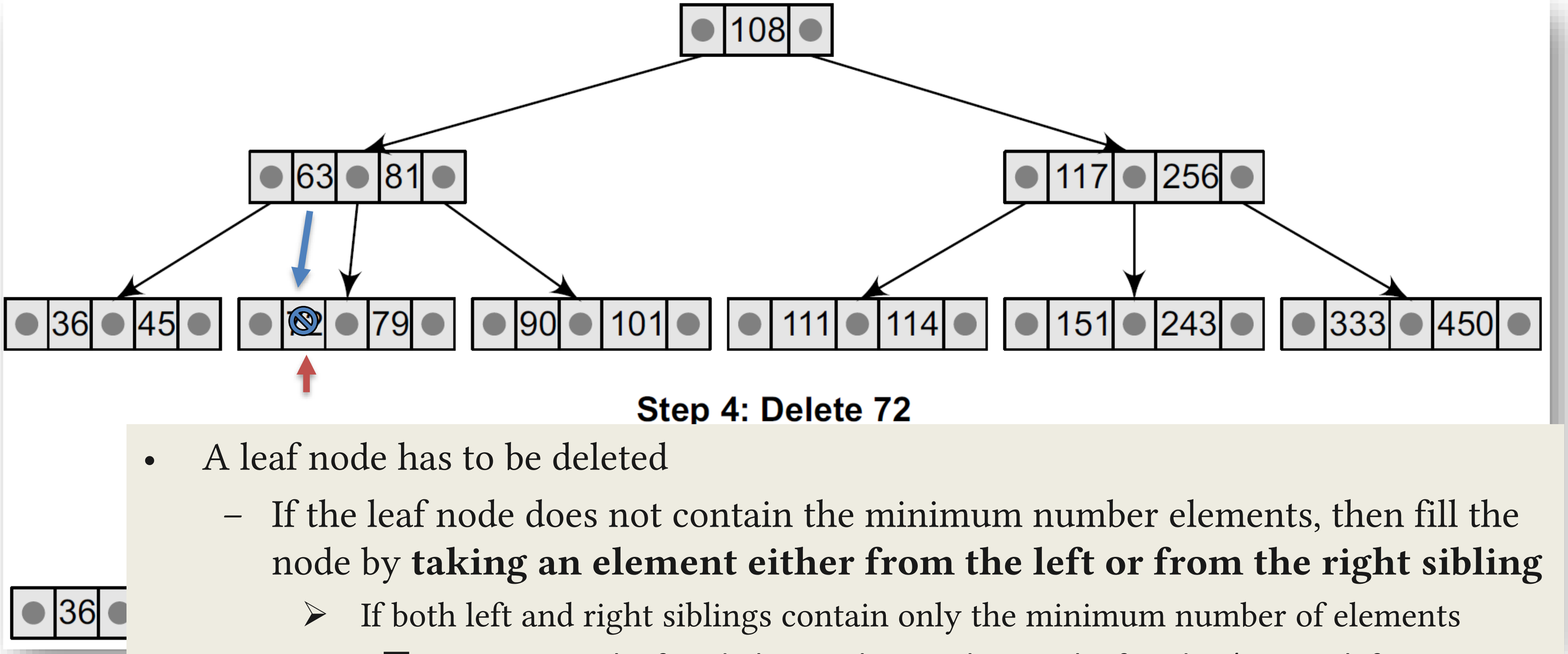

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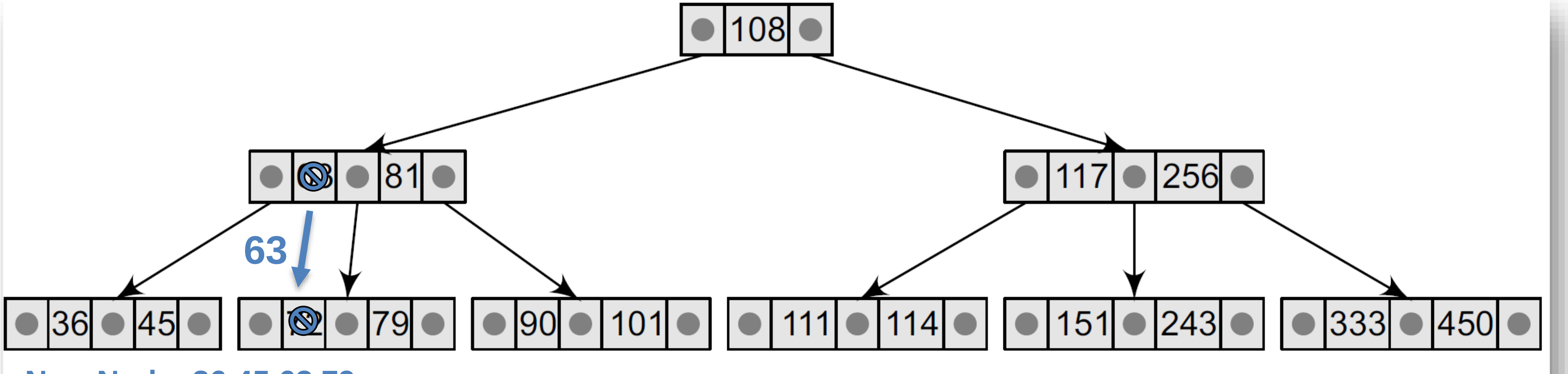

New Node: 36,45,63,79

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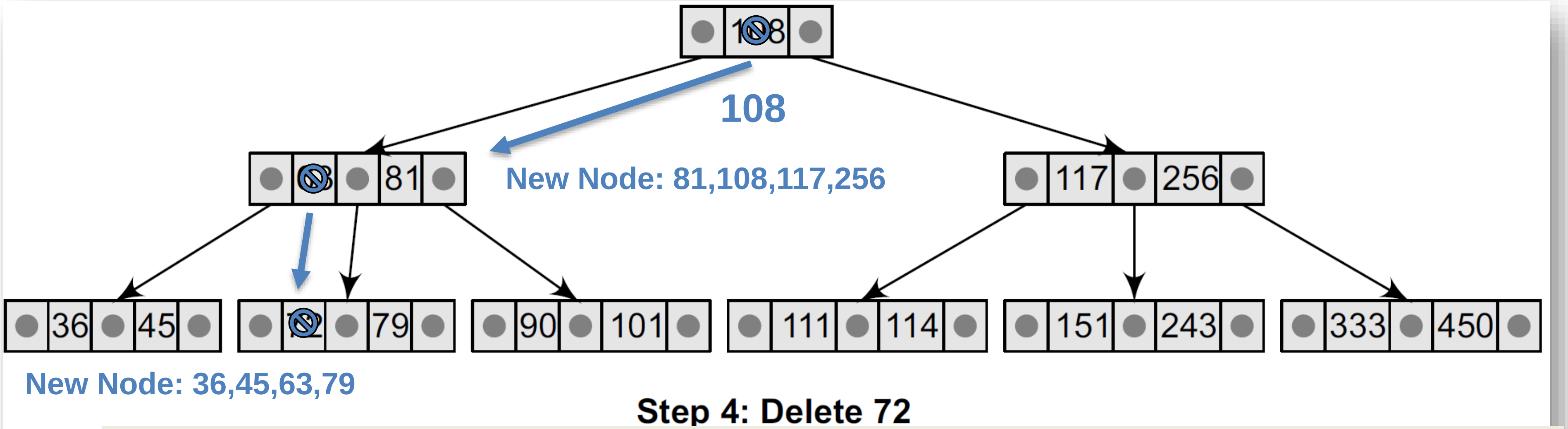
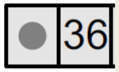


108

New Node: 81,108,117,256

New Node: 36,45,63,79

Step 4: Delete 72



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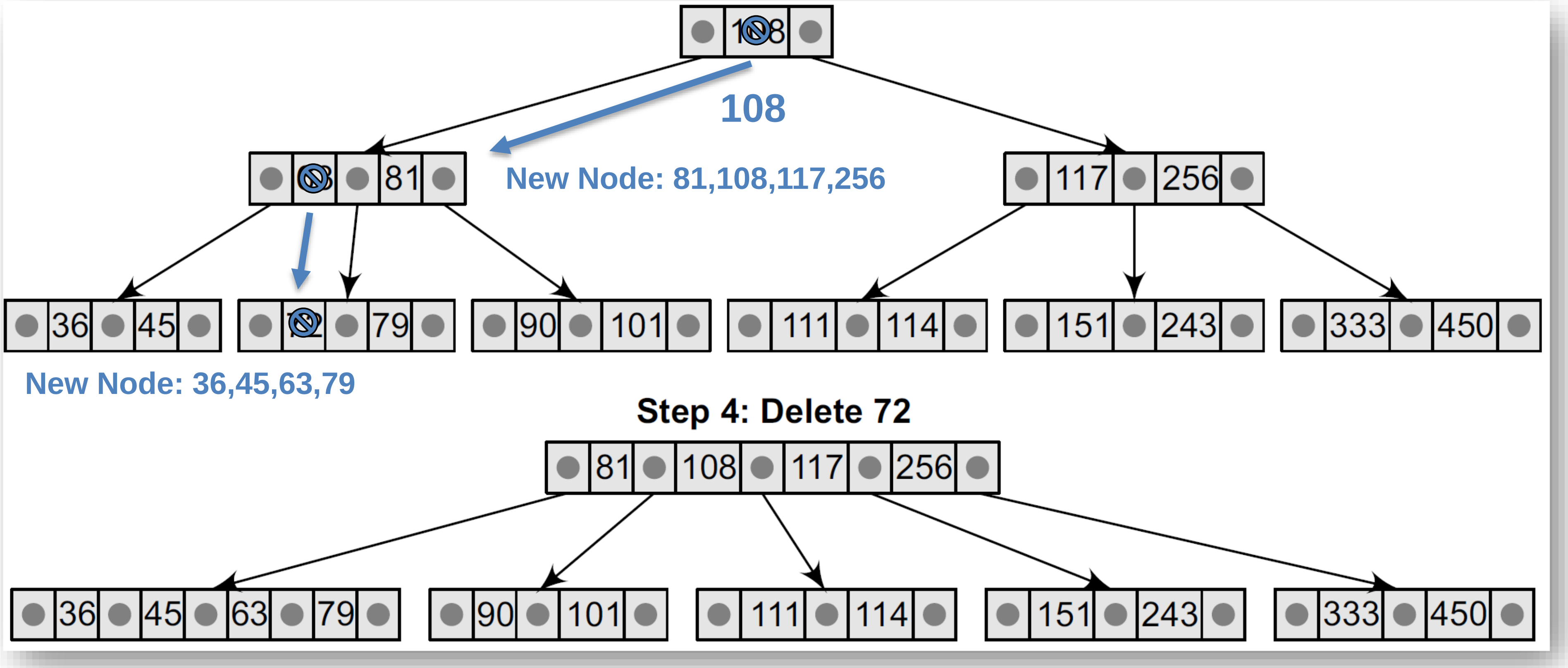
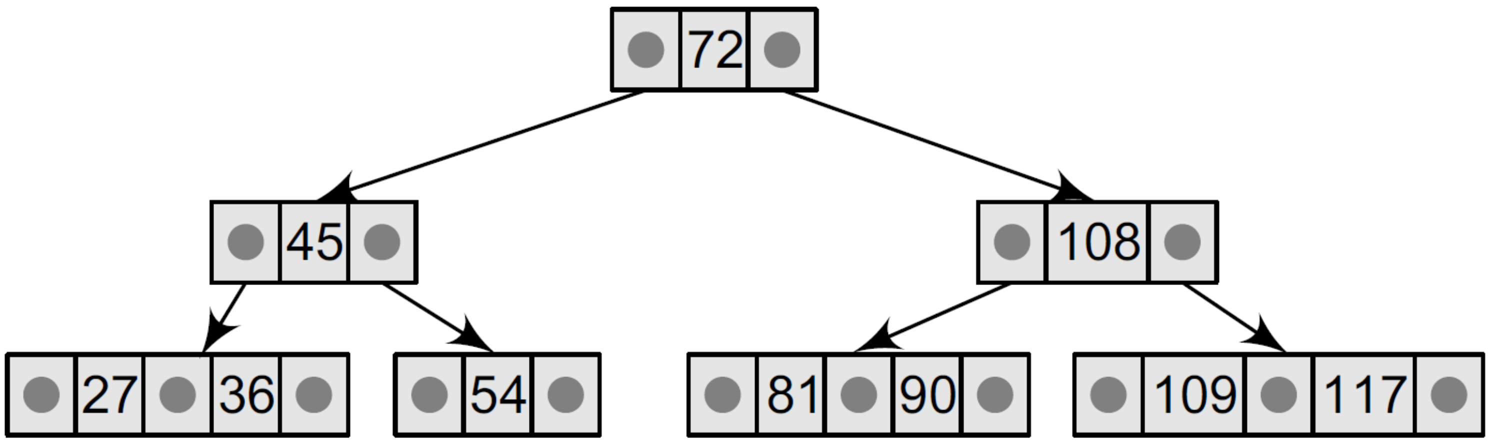
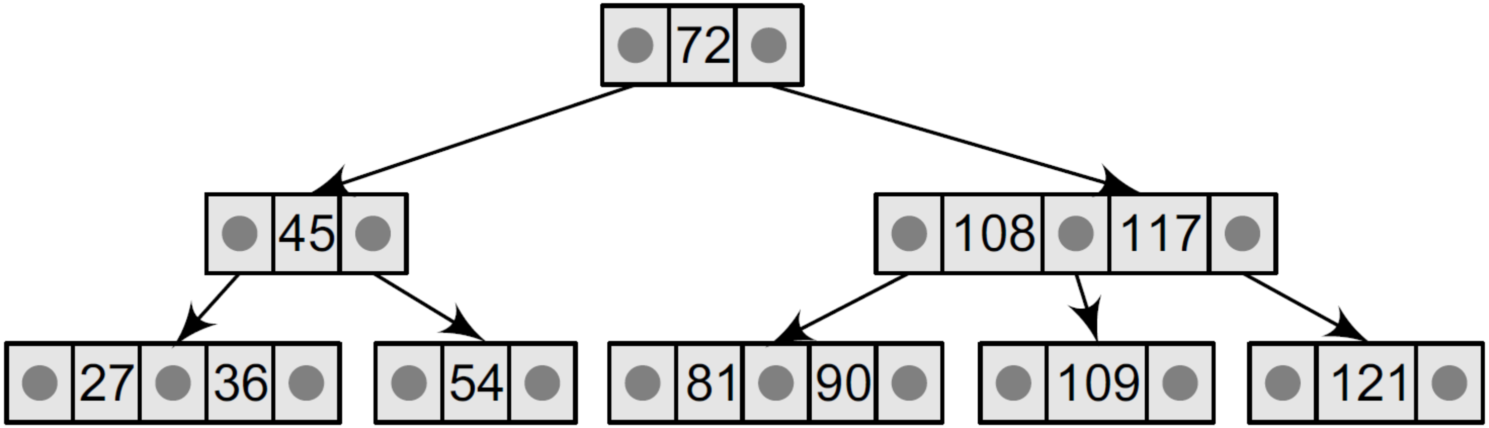


Example – 2.

- Given a B tree of order 3, please insert 121, 87 and then delete 36, 109



Step 1: Insert 121



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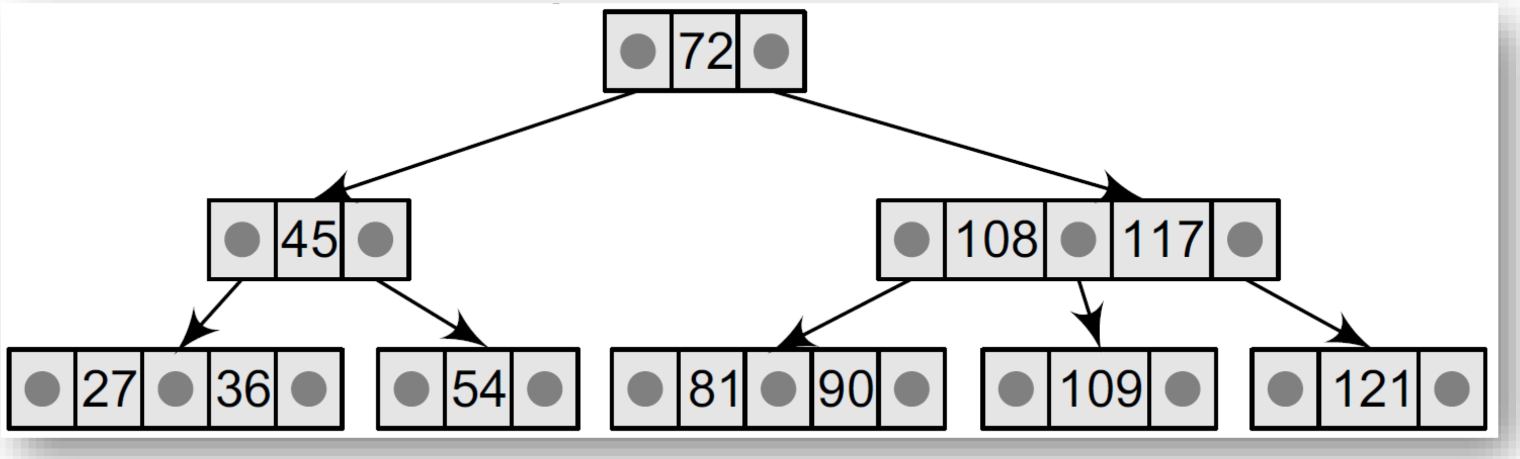

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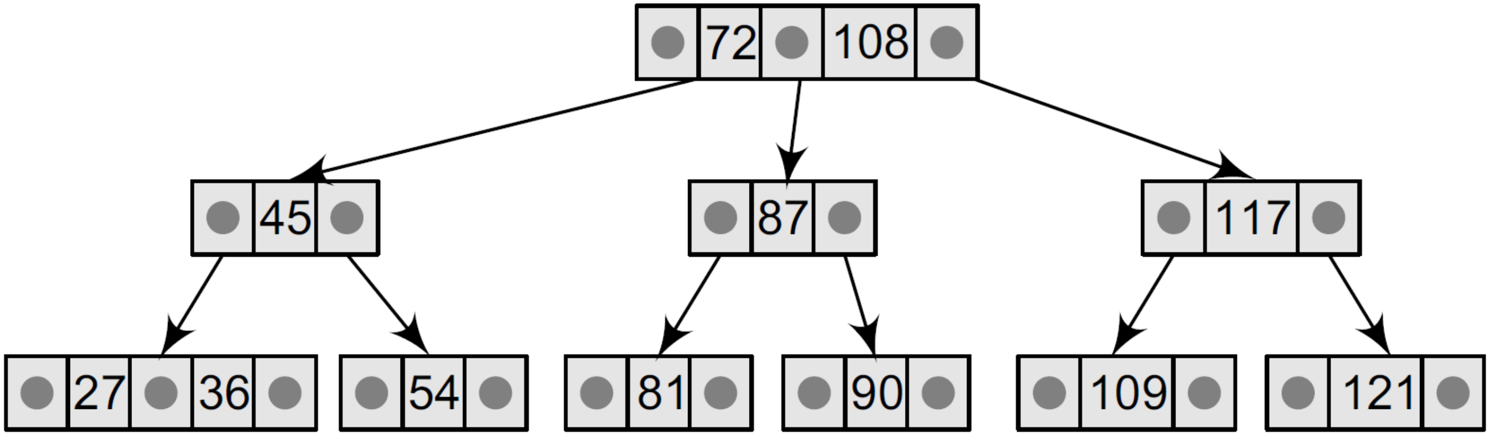

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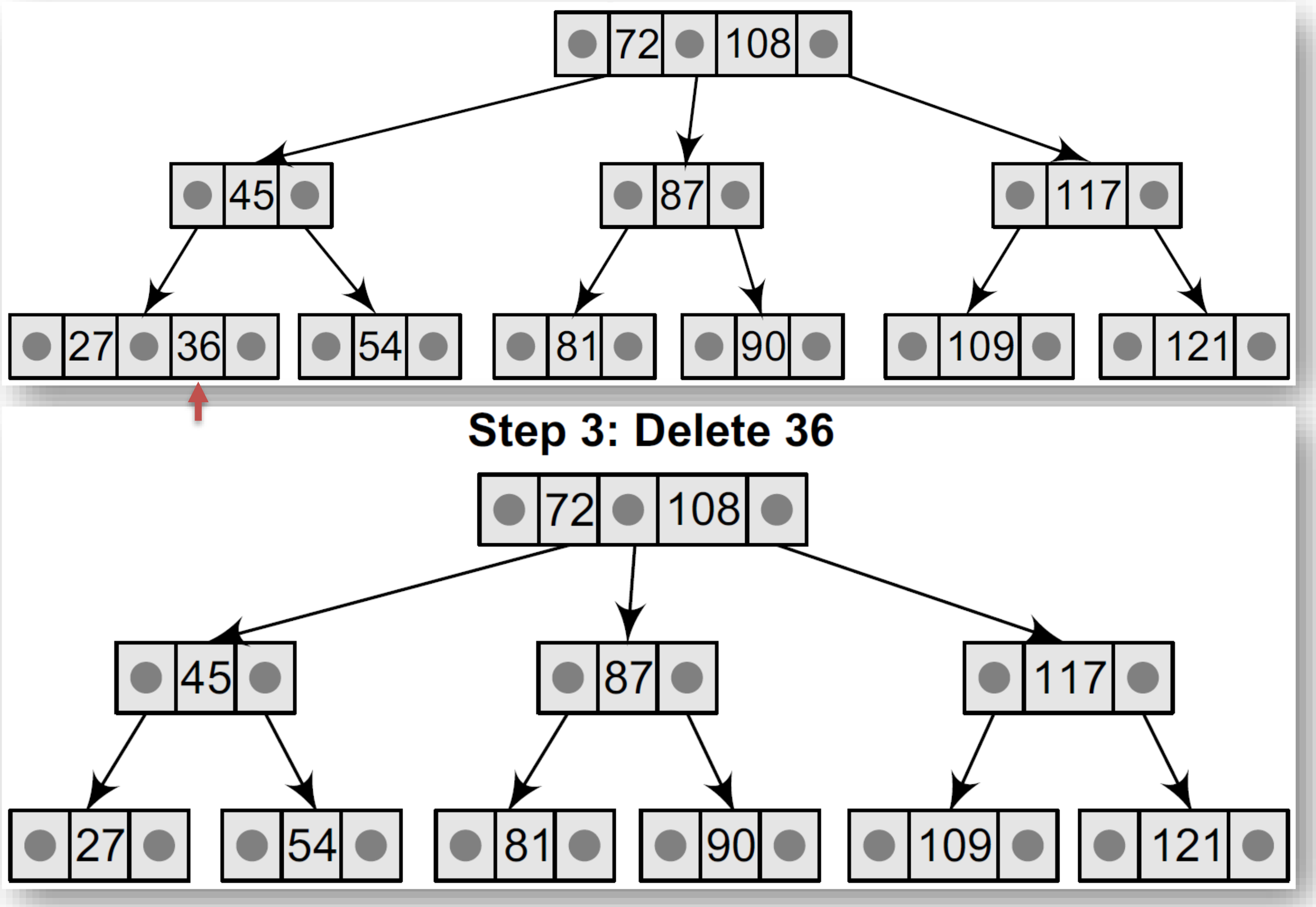

Step 3: Delete 36

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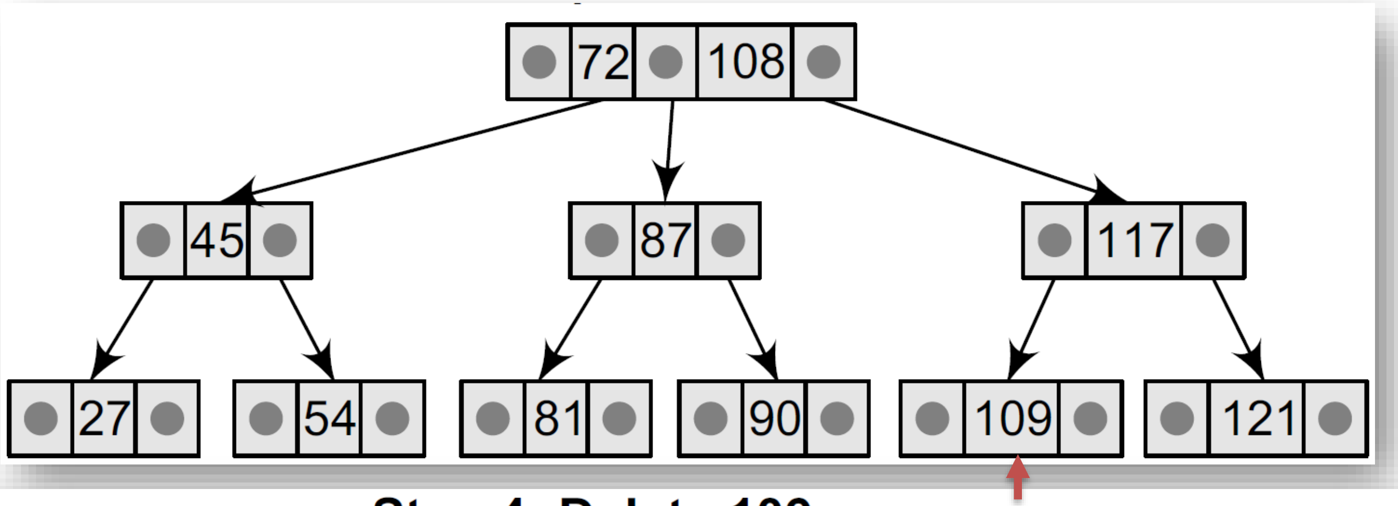

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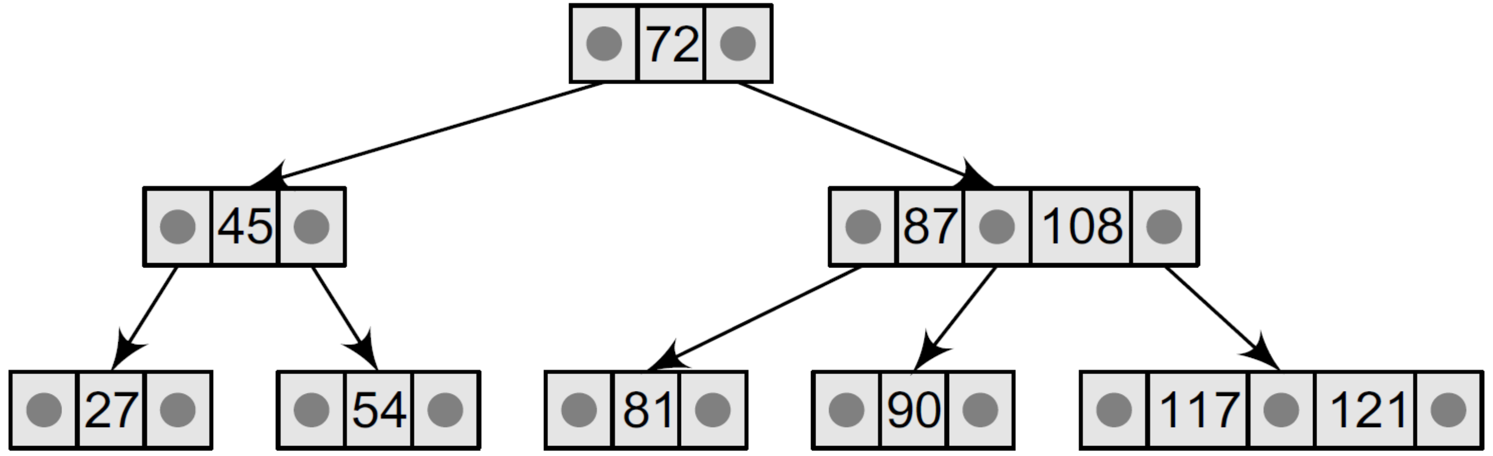

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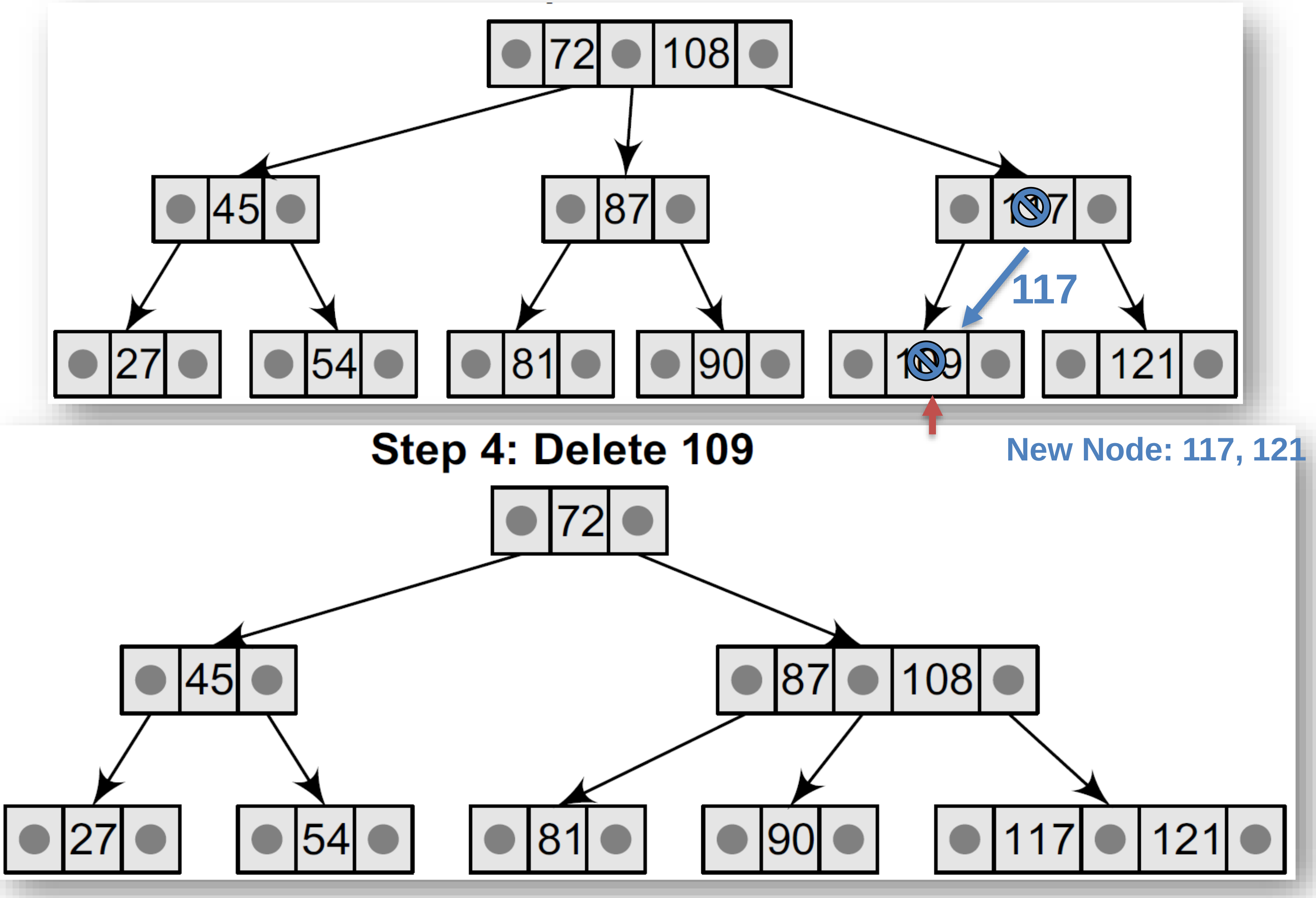

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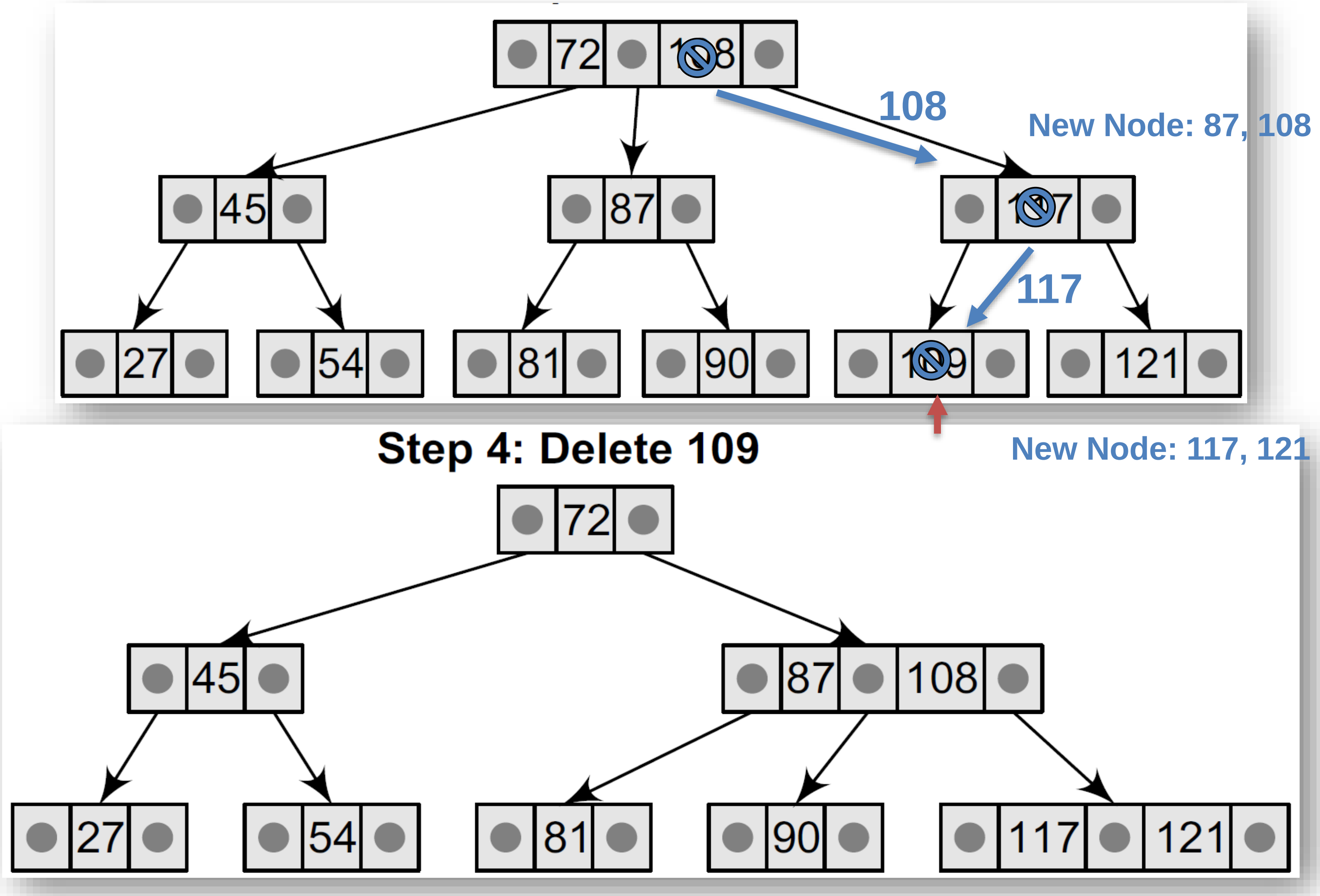
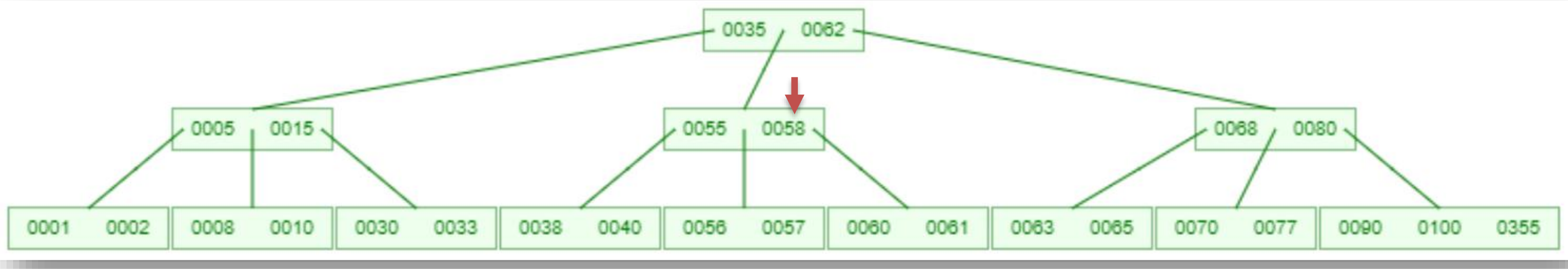
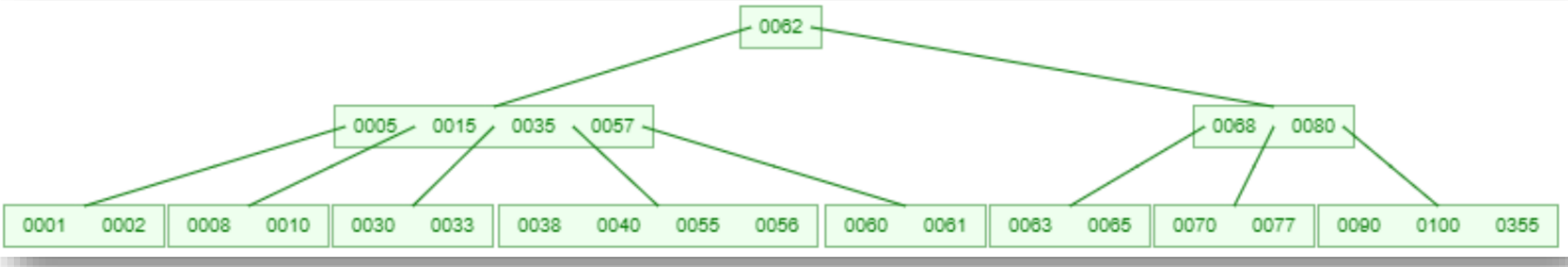


Example – 3.

- Given a B tree of order 5, please delete 58, 65





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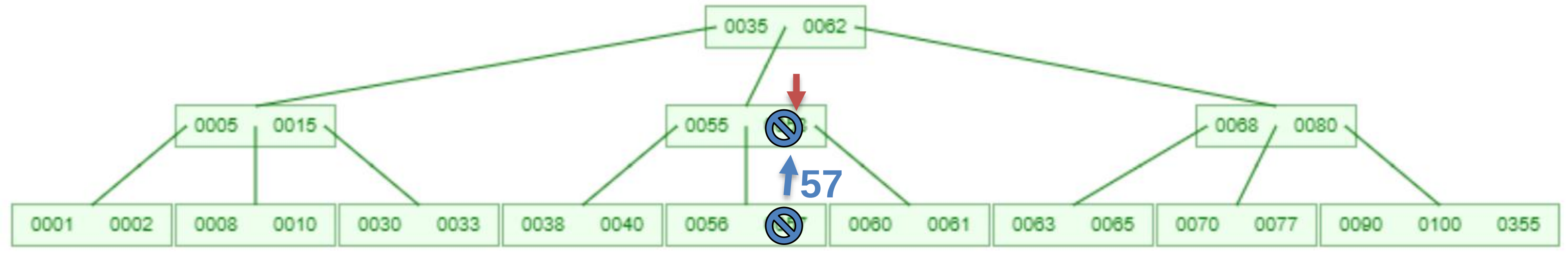

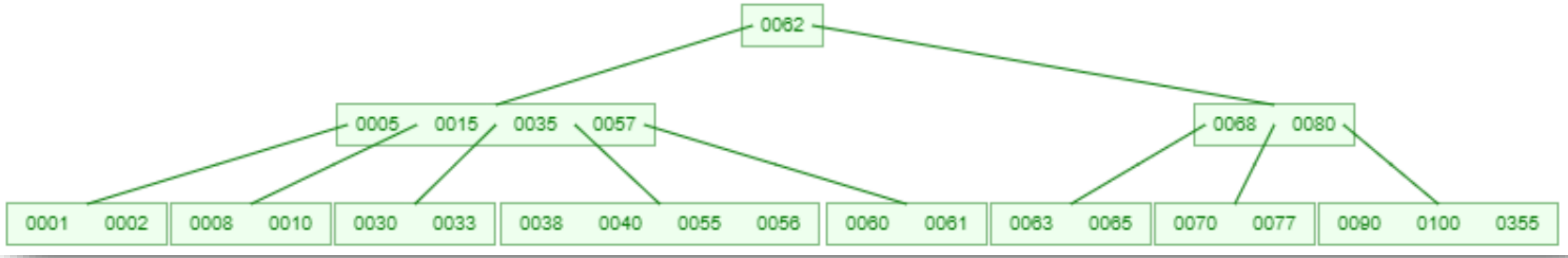

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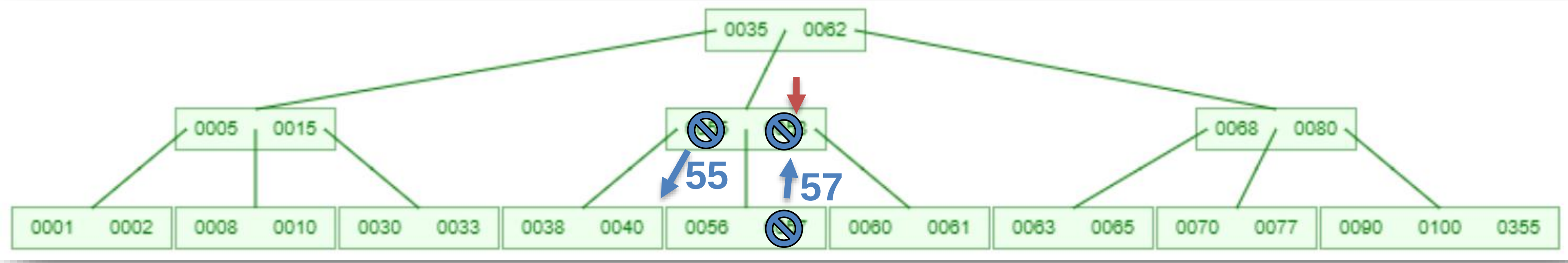
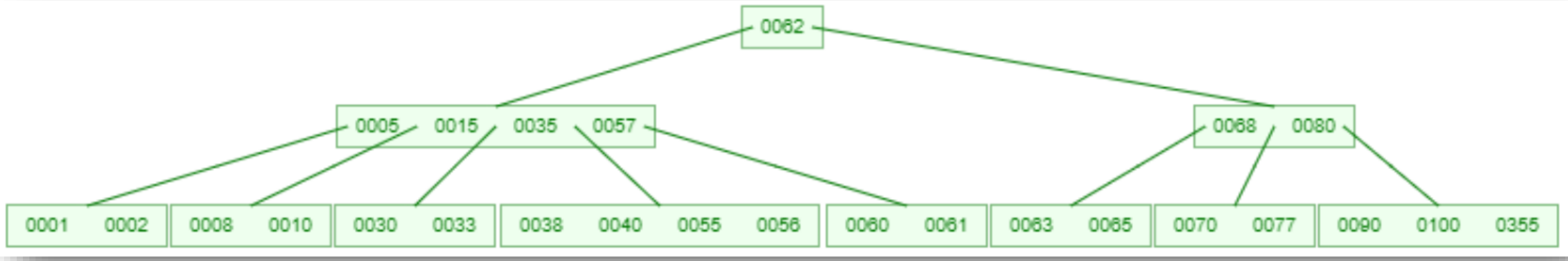


New Node: 38, 40, 55, 56



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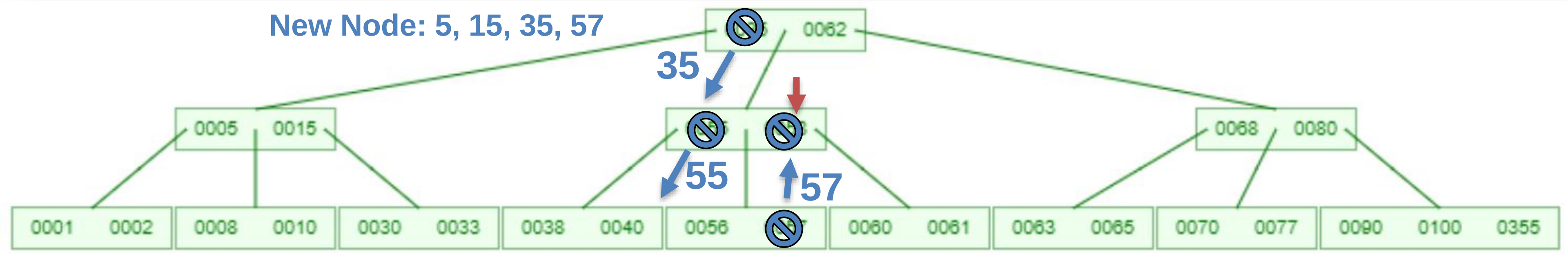
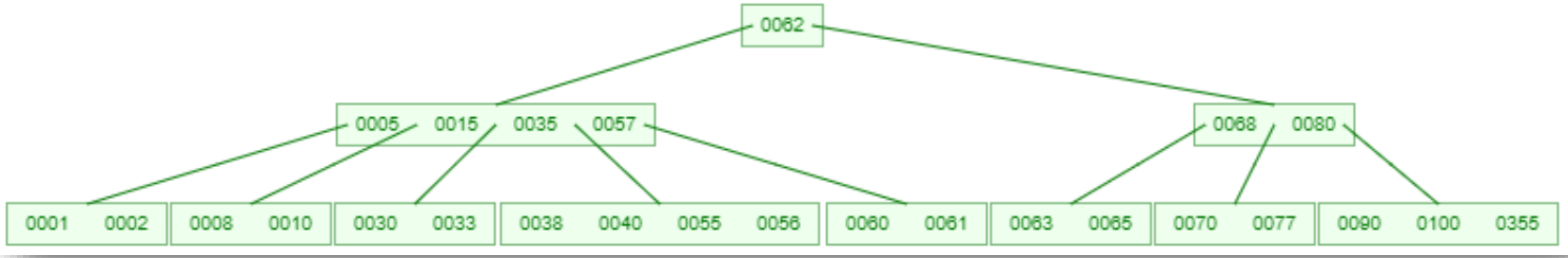

New Node: 5, 15, 35, 57

New Node: 38, 40, 55, 56



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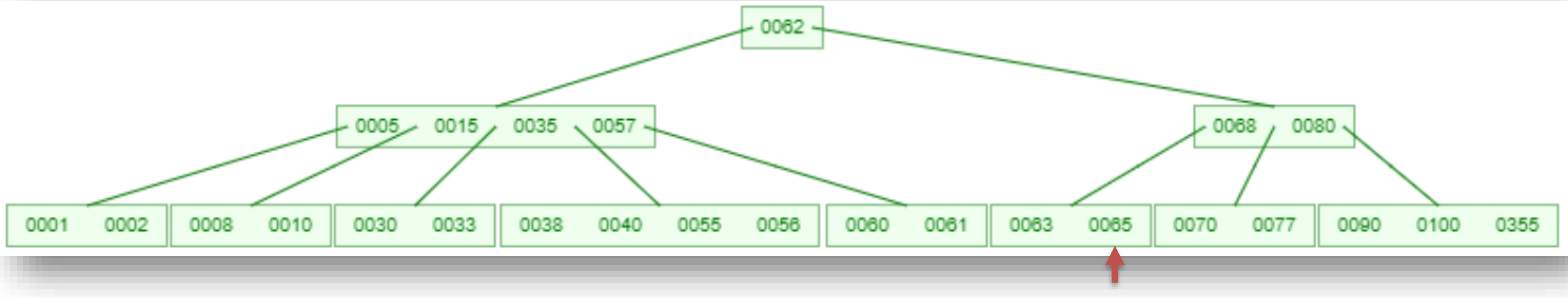

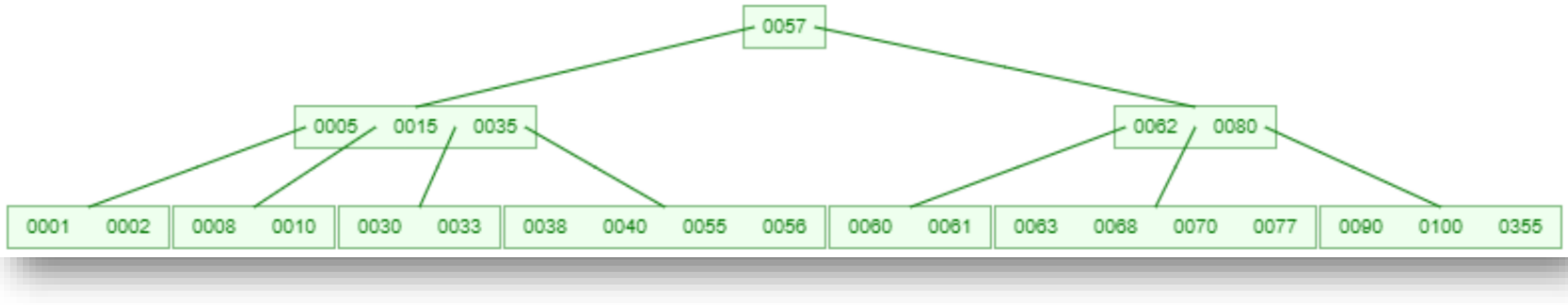

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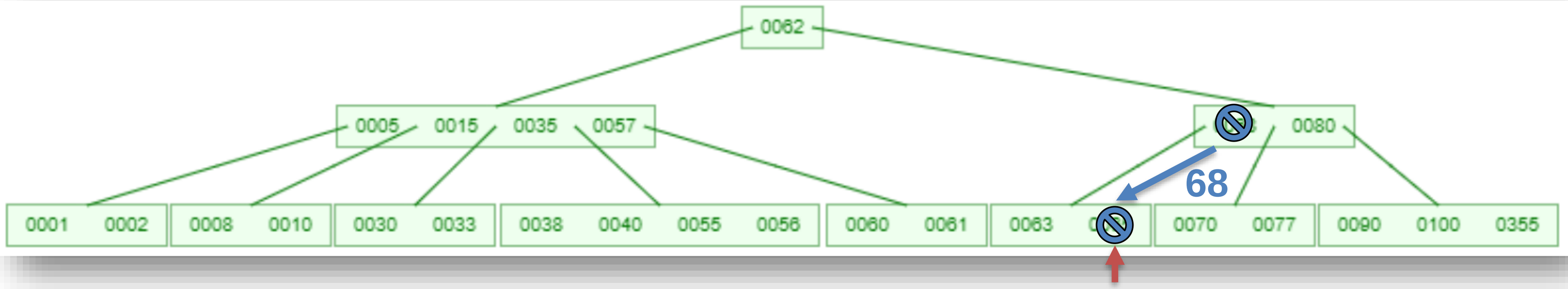
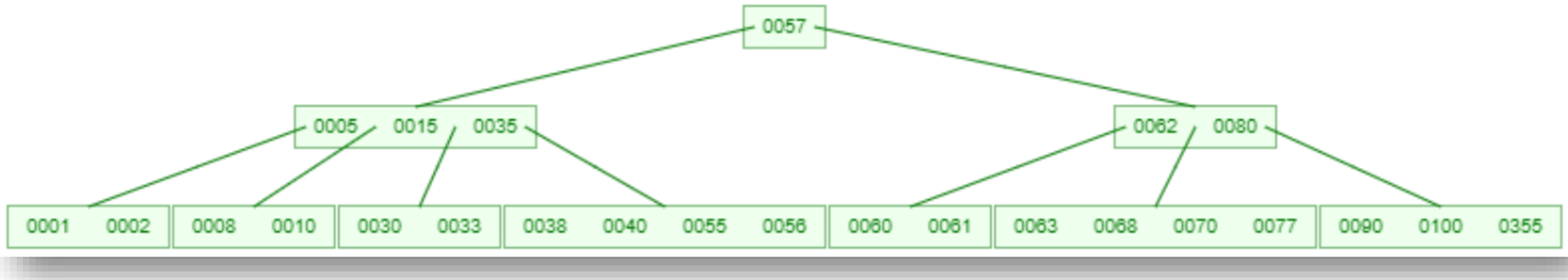


New Node: 63, 68, 70, 77



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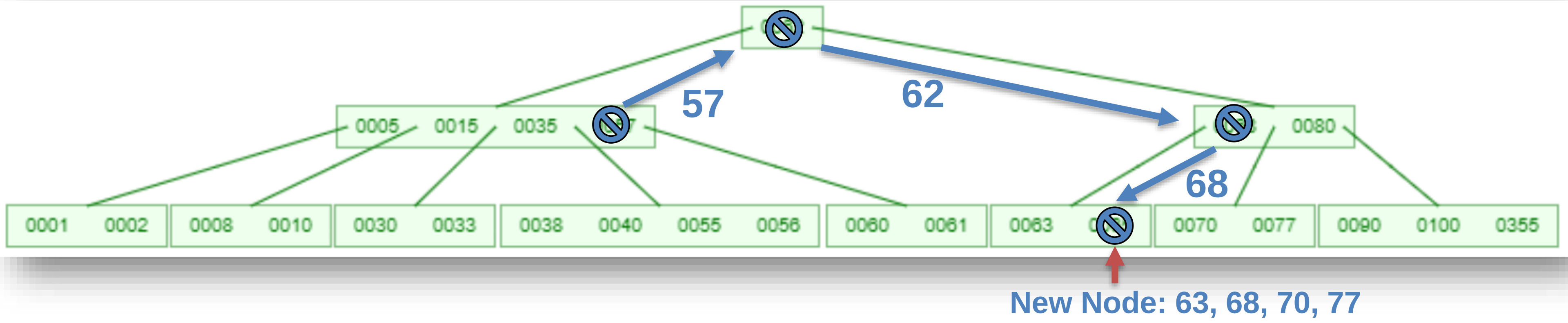

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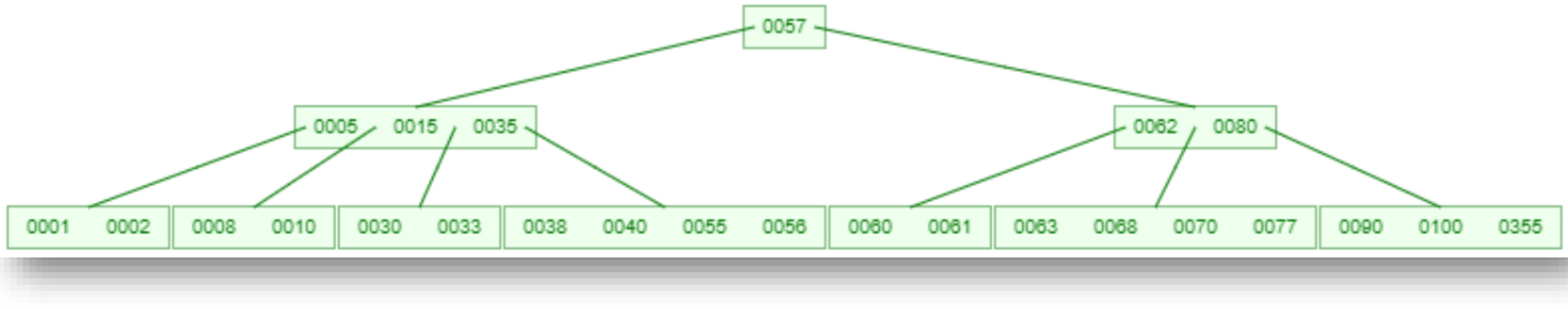

Check the Demo!

- https://www.cs.usfca.edu/~galles/visualization/BTree.html

Questions?

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